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La Voûte de LeFevre

A Variable-Volume Compression-Only Vault

LeFevre拱

一个可变体量的仅受压拱

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质点弹簧系统通常被用于发展受压找形系统。本文提出使用质点弹簧系统去响应理想形式，从而生成由体积材料加工而成的可变体量仅受压结构。通过改变系统的深度和体量，通过材料深度被重新定向，生成与结构意义上的最佳形式（均匀厚度）相反的理想形式。本文阐述了如何生成、建造和测试由可变体量单元组成的仅受压拱。这一研究将推动物理体积计算及体积建造方法的发展。

Particle-spring systems are commonly used to develop compression-only form finding systems. This paper proposes to use a particle-spring system in response to a desired form in order to generate a variable-volume, compression-only structure fabricated of volumetric material. By varying the depth and the volume of the system, loads can be re-directed through the depth of material in order to result in a desired form, as opposed to a structurally optimal form that assumes a uniform thickness approach. This paper proposes to generate, build, and test a compression only vault composed of variable-volume units. This research will progress knowledge surrounding volumetric physics calculations as well as volumetric fabrication methodologies.

薄壳型仅受压结构系统对建成环境来说是相对陌生的；而从另一方面看，仅受压结构又是古老的。薄壳结构设想出一种最小而一致的截面，这一设想受到材料效能的驱动，表现为完全受结构因素（通常是重力）影响发展出的形式，因此，“找形”一词应运而生。建筑必须对结构问题做出响应，但同时也不得不解决一系列其他问题，如声学、形式、功能等等，因此，形式不一定全然受结构要求的驱动。例如，哥特式大教堂的石材，其横截面在不同深度均存在推力向量。这些大教堂并非依据理想的悬链线确定形式，而是在双重考虑了仅受压原理和建筑需求的情况下形成的。受这一方法的启发，本文探讨了通过由可变体量构成的仅受压系统生成理想形式的潜力。

Thin-shell compression-only structural systems are relatively new to the built environment. Compression-only structures on the other hand are ancient. Thin-shell structures assume a minimal and consistent cross-section. This assumption is driven by material efficiency. The results are forms developed exclusively by structural concerns (typically gravity), hence the term form-found. Architecture has to respond to structural concerns, but it also has to address a variety of other issues—acoustical, formal, programmatic, etc. It is not necessary for form to be driven strictly by structural requirements. For example, Gothic Cathedrals contain the thrust-vector within the variable depth of

我们已对现有的可变深度结构进行了很多研究，分析在材料深度中是否存在着推力向量，[1]其他研究方法假设采用固定深度材料来生成设计。本文所采用的研究方法是构想出一个理想形式，并允许可变体量对推力向量进行重新定向，从而生成基于结构和形式的可用性设计。如果说通常的研究追求的是结构的轻薄，那么本文追求的则是结构的形式，通过结构来解决建筑问题。本文并不是提倡复古，而是运用先进的途径与方法将那些早已被遗忘的知识重新植入建筑语境之中。

系统、结构和形式响应

质点弹簧系统以连接到线性弹簧的集中质量（质点）为基础。本研究所使用的解算器是西蒙・格林伍德实现的质点弹簧系统的一部分 [2]。“系统中的每个质点都有其位置、速度、可变质量、以及概括所有作用力的一个总向量。” [3] 龙格-库塔解算器并不需要生成悬链线或是荷载分布，但需要用于评估不规则分布荷载。研究假定生成的几何形式并不是理想的悬链线，因此这一方法将多用于分析不规则的荷载分布。基于对弹簧质点系统的探索，前人已经创造出一些虚拟找形方法，如阿克塞尔・克利安的CADenary软件 [4]。

只要推力向量存在于系统横截面中段三分之一范围内，仅受压结构就能够站立。尽管我们并不总能预知一个结构是否会倒下，但我们能够预测其是否能够站立。有一篇论文[5]介绍了砌体结构的安全定理，安全定理表明：只要能够在结构的截面范围内能够找到一组处于平衡状态的压力网络，那么仅受压结构就能保持站立。这一方案有可能是下限解，因为当评估现有结构时，我们并不能总是获知力网的确切位置 [6]。而本文所采用的方法则能够计算并确保在材料厚度范围内存在着推力向量 ①。

弗雷・奥托[7]和安东尼・高迪等研究学者制作的找形模拟模型，或是阿克塞尔・克利安开发的虚拟找形工具CADenary [8]，都证明了要控制和预测最终找到的形式结果是有难度的。此外，如果找到的形式和模型之外的作用力不相匹配，我们也很难将两者结合为一个方案。因此，本文提出了形式响应策略。在一端输入理想形式，生成可变体量的解决方案，以便外部作用力和基于解算器的模型之间能够发生交互作用。

方法论

LeFevre拱是由基于解算器的模型计算所得，模型从不完美几何体中生成仅受压结构，它需要一个确定的几何体作为输入端，然后在几何体上开口以改变每一单元的质量。这一动态系统依据体量计算重新配置了单元质量：假如单元A的体量为单元B的两倍，那么其质量相应为单元B的两倍。项目材料必须是连贯且实心的（空心材料不能运作）。正是由于项目的质量和体量分布，使得系统中永远保持着零张力，因此这一计算方法产生了能够“永远”站立的结构。

the stone's cross-section. These Cathedrals are not determined by idealized catenary form, but through a confluence of architectural desires with compression-only principles. With this approach as inspiration, this paper addresses the potentials of compression-only systems to be resolved through a variable-volume in order to obtain a desired form.

Much research has been done in analyzing existing variable-depth structures to determine if a thrust vector falls inside the depth of material [1] .Other methods assume a fixed depth of material in order to generate a design. The method proposed in this paper assumes a desired geometry and allows for a variable-volume to re-direct the thrust vector as a means to produce a viable design that concerns both structure and other formal concerns. If typically one assumes thin, this paper assumes form.

This method is dedicated to addressing architectural concerns with structural results. This paper does not advocate for the reversion to a past architecture. It promotes the insertion of lost knowledge into our current means and methods of making.

Systems, Structures and Form Responding

Particle-spring systems are based on lumped masses, called particles, which are connected to linear elastic springs. The solver used for this research is part of a particle-spring system implemented by Simon Greenwold [2]. 'Each particle in the system has a position, a velocity, and a variable mass, as well as a summarized vector for all of the forces acting on it.' [3] This Runge-Kutta solver is not necessary to generate a catenary (even load distribution), but it is necessary when evaluating an irregular load case. The method applied in this research will always be an irregular load case because it is assumed the resulting geometry is not an idealized catenary form. Particle-spring systems have been explored to create virtual form-finding methods such as Kilian's CADenary tool [4].

A compression-only structure will stand as long as the thrust vector of the system falls within the middle third of its cross section. It is not always predictable that a structure will fail, though it is possible to know if it will stand. A paper [5] introduced the safe theorem for masonry structures. This theorem states that a compression-only structure can stand so long as one network of compression forces can be found in equilibrium within the section of the structure. This solution is a possible lower-bound solution. When evaluating existing structures, it is not always possible to understand where exactly this force network is [6]. The method applied in this paper can calculate and assure a thrust vector falls within the thickness of material ①.

LeFevre拱的柱体细节
Column Detail, Matter Design, La Voûte de LeFevre Banvard Gallery, 2012

① 针对无筋砌体结构的下限分析的更多资料，请参见Heyman 1982、 Huera 2001及 Huera 2004。
For further reading on lower-bound analysis of unreinforced masonry structures, see Heyman 1982, Huera 2001, and Huera 2004.

基本几何体

本文假设基本几何体是由模型的外在因素（声学、形式、建筑规范等）预先确定的几何体。未来的研究将关注如何使结构需求和其他形式的驱动因素之间的关系更加流动和交互。尽管几何形式并不一定严格地从结构需求出发，但形式与结构之间的关系必须相近才能生成理想的解决方案。在之前的计算测试中 [9]，以大多数任意几何体作为输入端，计算都能顺利运作；而体量可变的计算则更为微妙，它需要将一系列的数值输入到系统中去，包括上下边界面。这些边界面将单元深度参数化，使其在形式生成的过程中是可变的；而在可变体量的计算中，则是固定的；同时，可变体量的计算要求各单元之间的节点都应位于系统之内。这些质点均匀地分布在上下边界面之间的基本几何体上，引入质点弹簧系统，在表面进行点的定位和分布——越接近几何体的上部，点与点之间的距离越大。

质点弹簧系统

质点弹簧系统由大量质点、连接质点的弹簧以及质点上不断产生反馈到系统的作用力组成。尽管组成方式保持不变，但系统已重组出各种方案 [10]。本文采用的是上文提及的均匀分布系统。

垂直距离 VS 体积

分析砌体拱的常用做法是利用静态区块分析将一个拱分解为数个多边形。每一多边形的面积决定了垂直推力向量 [11]。先前的迭代计算通常采用高精度的垂直距离为每一质点赋予新的相对质量，而本文则采用不同于面积或距离的体量来决定单元的质量。目前已有类似研究，通过体量来分析和决定某种结构的可行性，[12]本文则利用体量的可变性来确保方案的可行性。质点的位置决定了虚拟推力网络的位置。为了生成方案，这些质点需要在计算过程中不断地移动，直至达到平衡状态。一系列操作发生在计算的每一次间隔，将计算复杂化，不再仅仅是简单的距离测算。新的质点位置生成了三维的泰森多边形，这一多边形同底部的基本几何面边界相交。在生成插值曲线的地方，曲线间相互交叉产生了点；与此同时，中心点（同样为质点）找到了位于上界面的闭合点，生成了垂直于两点连线的圆，该圆形所处的平面成为计算数控机床操作的边界面，作为有效的建造限制。圆形和曲线经过放样创造出曲面，该曲面在系统中被削减。这些曲面的交叉部分挤出至上一表面的最近点，创造出拱顶中作为离散单元的楔形块 ②、③。每一单元包含一个封闭体量，告知整个系统其相对相邻体量的质量。

Form-finding analog models by such researchers as Otto[7] and Gaudi, or even the virtual versions like Kilian's CADenary[8] have proved it is difficult to control and predict the results of the final found-form. Moreover, if that form does not correspond with a force that is external to the form-finding model, it is difficult to resolve the two into a solution. This paper proposes form-responding as approach. Form-responding takes a desired form as input and produces a variable-volume solution to allow for interaction between these external forces and the solver-based model.

Methodology

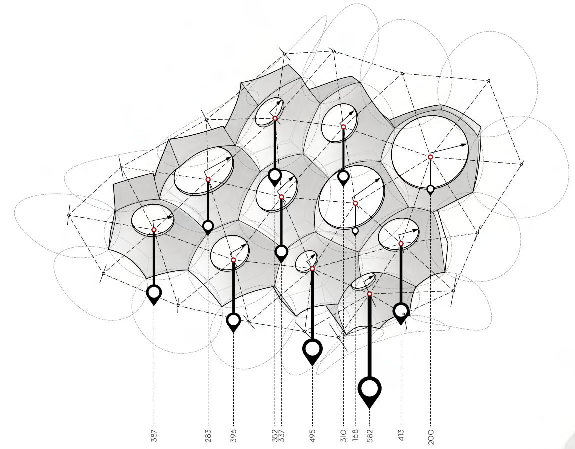
The vault is computed with a solver-based model that elicits a compression-only structure, from a structurally non-ideal geometry. The model requires a fixed geometry as input, and opens apertures in order to vary the weight of each unit. This dynamic system re-configures the weight of the units based on a volumetric calculation. If unit A contains twice the volume of unit B, then unit A weights twice as much. It requires that the material of the project be consistent, and solid (hollow does not work). The computed result produces a project that will stand 'forever' as there is zero tension in the system precisely because of the weight and volume of the project, and not in spite of it.

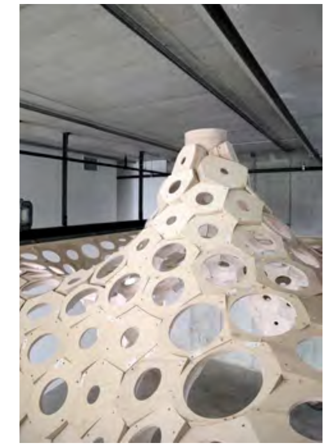
Base Geometry

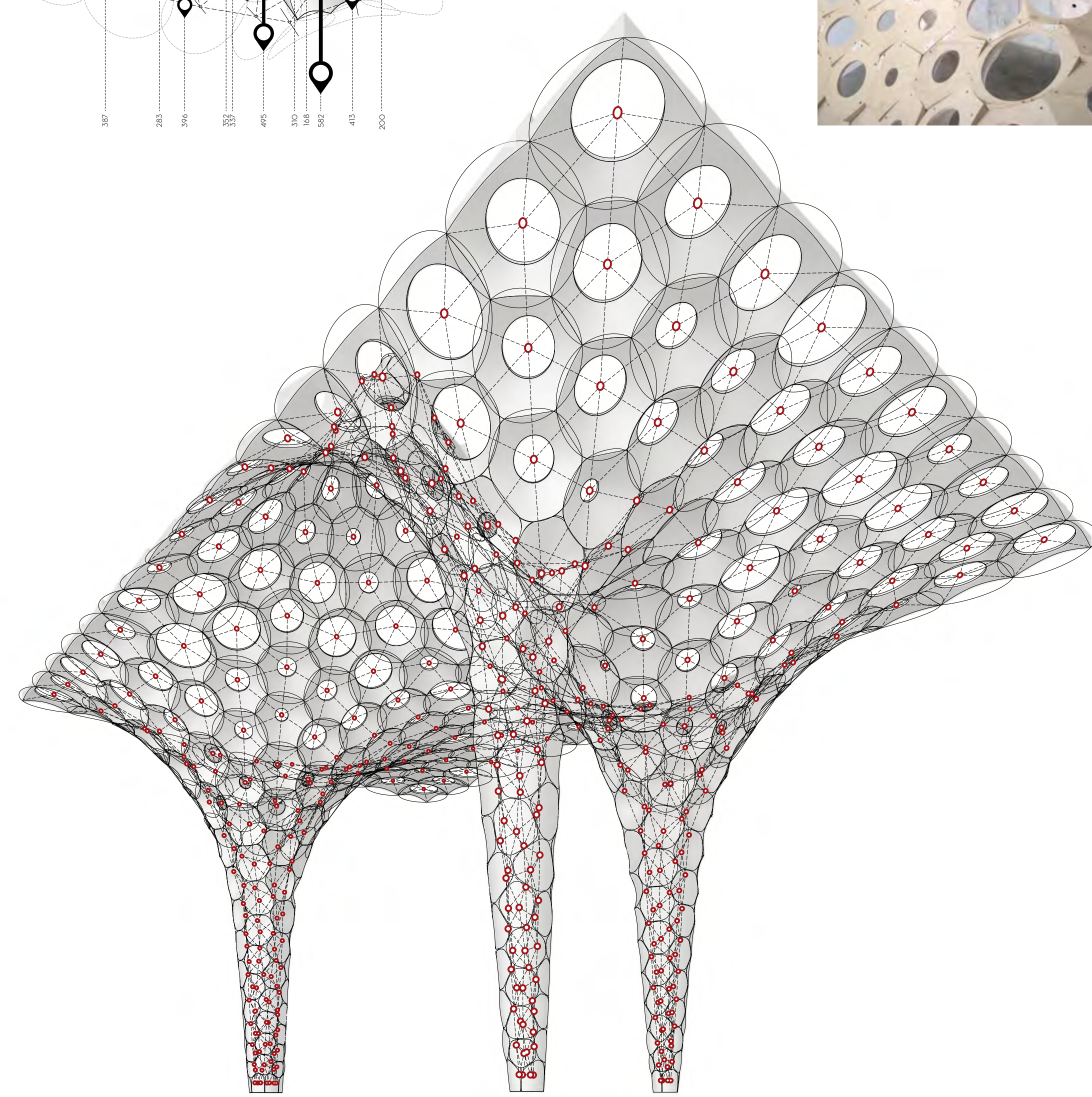
This paper assumes the base geometry as fixed. The assumption is that this geometry has been pre-determined by a force external to the model — acoustics, formal, building-code, etc. Future research could allow for a more fluid and reciprocal relationship between the structural requirements and these other formal drivers. While this geometry is not strictly aligned with structural concerns, it must be close in order to result in a solution. In previous versions of the calculation [9] almost any geometry as input would work. The Variable-Volume calculation is more nuanced. This calculation requires a number of inputs to the system. It requires both an upper and lower bound surface. These surfaces parameterize the depth of the units as variable during the form generation, but fixed during the variable-volume calculation. The calculation also requires a location for the node of each unit to be located within the system. These particles are evenly distributed across a base geometry that falls between the upper and lower bound surfaces. This distribution employs another particle spring system to locate and distribute the points across the surface increasing in distance from each other as they approach the upper elevations of the geometry.

② 楔形块，通常是石材，建造拱门或拱顶时采用的一种楔形构件。
Voussoir: a wedge-shaped element, typically a stone, used in building an arch or vault.

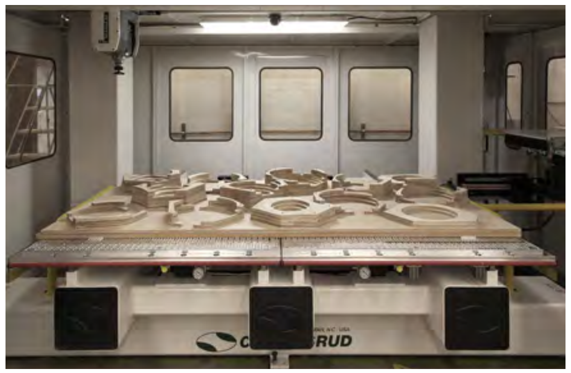
③ 考虑到与建造方法的相互关系，将包裹体量的表面几何设置为直纹曲面。
The surface geometries enclosing this volume are generated with ruled surfaces due to a reciprocal relationship with the method of fabrication.







上图说明了可变体量计算结果——位于拱顶的单元随高度升高而逐渐变大，位于立柱的单元随高度降低而逐渐变小。这三类输入信息成为弹簧质点系统重新计算的数据基础。系统将持续进行计算直到找到平衡状态并生成解决方案。
These figures demonstrates the result of the variable-volume calculation—an enlarging of the units in the vault, and a tightening of the units down in the columns. These three inputs serve as the datum with which the particle-spring system computes it-self against. These operations are calculated continually until the system finds equilibrium and a solution can be detected.



等待五轴CNC铣削加工的能组合成理想几何体的粗略聚合构件（上图）。密歇根大学建筑与城市规划学院FABLab工作室的建造支持（下图）。
Roughed aggregated blanks of the desired geometry await the milling operation on the five-axis machine (above). With fabrication support by the University of Michigan Taubman College FABLab (below).

设计

本项目有意实现了从柱到拱的拓扑学转化 ④，这种转化似乎无任何中断，但事实却并非如此。实际上，柱与拱之间有所差异，柱是被视作单元结构的实心体；而拱则被拆分为多个构成单元 ⑤。我们试图进行单元间的无缝连接，但木材的纹理暴露了真相。这一虚假的真实世界存在很好的理由去解释：柱并不能扮演拱的角色，柱中的推力向量是垂直的，并非渐渐水平向的。因此，柱不能抵抗水平推力，而能够抵抗纵向弯曲力。柱的主要考虑因素是坚固性。

从实心柱向分散拱的转化，其差异可以通过修饰得以解决。独立单元的修饰延续到立柱上，仿佛柱子是独立单元向地面的延伸。这种修饰不是组成拱顶的锥形布尔几何体的简单延续，而是采用了一种新的类似的方法。它参考了锥形布尔几何体，但拒绝对其进行简单的复制。几何的变化使得系统不仅能够调整体量（如拱中的应用），也能够促使从分散到光滑的过渡。随着单元向柱底延伸，它们变得越来越小，但低凹处却慢慢地向柱表面靠近，创造出连续性的错觉，直至将连续体推至基础。这一符号表明，受上方拱顶的重量影响，柱子不得不向外凸出。

Particle-Spring System

The particle-spring system is composed of a number of particles, the length of the springs that connect the particles, and the continual resulting forces on each particle informing the system. While the organization is consistent, the system has been reconfigured in a variety of solutions [10]. This paper employs an evenly distributed system as described above.

Vertical Distance Versus Volume

When analyzing masonry arches, it is common practice to use static block analysis to break down an arch into a few polygons. The area of each polygon determines the vertical thrust vector [11]. Previous iterations of this calculation employed a high resolution of vertical distances to inform each particle with its new relative weight. This paper employs volume as opposed to area or distance. Similar work has been conducted using volume to analyze and determine the viability of a structure [12]. This paper employs the variability of the volume to ensure a solution.

建造

LeFevre拱由波罗的海桦木胶合板制成，胶合板由19.05mm厚的薄板加厚而成。或许这证明了这个行业的现状：获取体积材料十分困难。通过对薄板的数字切割，将每个定制单元切割为这种厚度，之后经过物理重组，生成最终几何形式的粗略体量。将粗略体量编号排在一大块板上，经过胶粘、真空压缩，重新放置于数控机床上。尽管这一流程更费劲，但它比将一整块实心材料直接雕刻成形更加节约材料。

该项目在五轴铣削机上进行，使用的削屑工具致力于花最小的力气切割最多的材料。该路径不使用工具头的末端进行工作，而是使用侧刃铣削 ⑥ 来移除尽可能多的材料。不同于菲利贝尔·德洛姆 ⑦ 的点追踪法，这一方法通过线来追踪几何形态，因此它要求单元由直纹曲面构成。尽管该限制条件放宽了对立柱的要求（一种更为典型的曲面铣削操作能够产生修饰性的凸起），但是其体现了拱顶的锥形布尔几何体为项目的一部分。工具操作的转化同样说明了对立柱及拱顶差异的理解。

分析

本项目采用假设的零填充方式进行建造。因为设计要求拱必须是可拆卸的，因此建造中并不采用砂浆。因为零容差的方法，差异、错误及缝隙不可能得到解决。为确保一些难操作位置的现场施工，团队采用了手工带锯移除问题单元背面的碰撞材料。现场雕刻并不会影响单元的边缘，但却在楔块表面留下了不一致的缝隙，这个缝隙和印加楔体 [13] 流程留下的缝隙恰好相符。泥瓦匠会从墙背面将砂浆填充至石头间的空心楔形中，而墙体前面（建筑表面）则是无砂浆的。关于印加楔体的利用潜力，未来仍有很大的探索空间。

The location of the particles defines the virtual thrust network. In order to ensure a solution, these particles are requiring to be moving during the calculation until they find equilibrium. At each interval of the calculation, a number of operations occur complicating the calculation beyond a simple distance measurement. The new location of each particle generates a three-dimensional voronoi calculation that intersects with the lower bound base geometry surface. This intersection then produces points at the intersection of each curve where an interpolated curve is generated. Simultaneously the centroid point (also the particle) finds the closes points on the upper bound surface and generates a circle perpendicular to the line connecting these two points. The plane this circle is generated on also serves as the flat backside that sits on the table of the computer numerically controlled (CNC) router, a useful fabrication constraint. The circle and the curve are then lofted with each other producing surface that is trimmed with the rest of the surfaces in the system. The intersection of these surfaces extrude to the closest position on the upper surface producing the voussoir② that discretizes each unit in the vault ③. Each unit now contains an enclosed volume that can inform the system with its weight relative to its neighbors.

Design

A deliberate attempt was made in this project to topologically ④ transition from column to vault. No break is inserted in this transition; however, this is a lie. In reality there is a difference between column and vault. The column is solid. It is treated as a single unit. The vault on the other-hand is discretized into its constituent units ⑤. This moment of discrepancy is attempted to be seamless; however, the grain of the wood demonstrates the reality. There is a good

④ 拓扑学：研究在变形（弯曲、拉伸、挤压，不含破坏）作用下不发生变化的几何物体的属性。
Topology: In mathematics, the study of the properties of a geometric object that remains unchanged by deformations such as bending, stretching, or squeezing but not breaking.

⑤ 在彼得堡大教堂中，采用了类似的从实心柱向楔形块的转化策略。这些楔形块在几何上边界未对齐，但在下边界（可视面）精确对齐。
TA similar strategy of the solid column transitioning into voussoirs above was employed in Peterborough Cathedral. These voussoirs also misalign on the upper bound geometry, while aligning precisely on the lower bound (visible surface).

⑥ 侧刃铣削是沿构件表面（如锥形脊侧壁）行进时使用立铣刀进行侧向切削的技术。
Swarf machining is a technique that allows side cutting with an endmill while proceeding along the surface of a part, such as the sidewalls of a tapered rib.

⑦ 菲力贝尔·德洛姆（16世纪）和帕拉迪奥一样，都是石匠的儿子。他在建筑领域为人们所熟知并非因为对形式或技术的深入了解，而是因为其建造师，或石匠的身份。在1567年出版的《第一本建筑》中，德洛姆介绍了艺术几何特征的方法和定义。该方法指导我们如何从图纸到建筑，及从建筑到图纸。因为这本书，德洛姆也被视为第一位专业建筑师。尽管，当代的建筑再现方式和德洛姆的作为建造方法模板的画法几何之间存在很大的区别，但他发明的技术使得当时的设计师与建造师之间能够进行指导和沟通。因此，德洛姆可以说是数字建造的鼻祖。
Philibert de L' Orme (Sixteenth Century) was, like Palladio, the son of a mason. He emerged into architecture, not through a series of rigorous understandings of form or technique, rather from the builder—or mason. In his printed work of 1567 Le premier tome de l' architecture, Philibert de L' Orme introduced the method and definition of art du trait géométrique. This method developed as a way to reciprocally draw what can be built and vice-versa. Because of this emergence, De L' Orme can also be credited as the first professional architect as his technique served to instruct and communicate between the designer and the builder, though an important distinction should be drawn between the representation of architecture we now generate and De L' Ormes descriptive geometry that served as method template to construct. In a way, De L' Orme can be considered the predecessor to digital fabrication.

结论

LeFevre拱证明了将围绕体量的当代建造方法和古代知识相结合的可能性。它成功地采用了物理模拟，借助体量计算和体量制作流程的协同，确保了结构的稳定性。在此案例中我们采用了聚合式波罗海桦木胶合板作为一种模拟，但我们仍旧能够看到其他体积材料，如蒸压加气混凝土、石膏或石材运用在这一领域的潜力。

拱的组装
Assembly of the vault

reason for this false-reality. A column does not perform in the same manner as a vault. The thrust-vectors inside the column are vertical, not progressively horizontal. To that end, a column does not resist horizontal thrust. It is resists buckling. The solidity of the column is paramount.

The discrepancy in transitioning from solid column to discretized vault is resolved via rhetoric. The rhetoric of individual units continues down the column as if the single and solid column was in fantasy an impossible continuation of the units to the ground. This rhetoric is not a simple continuation of the conical-boolean geometry that composes the vault. It is a new, yet similar approach. It refers to the conical-boolean, without repeating it. This shift in geometry allows the system not only to calibrate volume (as applied in the vault), but also to perform another transition from fragmented to smooth. As the units make their way down the column, they do get smaller, but the dimples slowly make their way to the surface producing the illusion of continuity, only to push through that continuity as the very base. This punctuation to the statement suggests that the weight of the vault above is so great that the column is forced to bulge outward.

Fabrication

The vault is produced with Baltic Birch plywood. The plywood is sourced in 19.05mm thick sheets awaiting the 'thickening'. Perhaps this speaks to the state of the industry that volumetric material is difficult to procure. Each custom unit is digitally dissected and sliced into these thicknesses, cut from the sheets, and then physically re-constituted into a rough volumetric form of their final geometry. These roughs are indexed onto a full sheet and glued, vacuum pressed, and re-placed onto the CNC router. This process is materially more efficient than carving these units from one solid block of material, though it is more laborious.

This project is produced on a 5-axis Onsrud router. The swarf [8] tool-paths utilized are dedicated to removing the most material with the least effort. Instead of requiring the end of the bit to do the work, this path uses the edge of the bit to remove much more material. Because this method traces the geometry with a line as opposed to point via Philibert De L'Orme's technique stereotomy [7] ,it requires the units be constituted of ruled surfaces. This constraint informed the conical-boolean geometry in the vaulted portion of the project, though relaxed in the columns where a more typical surface milling operation produces the rhetorical bulges. This shift in tooling operation also speaks to the understanding of difference between column and vault.

Analysis

This project was fabricated with an assumed zero-fill approach. As part of the requirement that the vault must be dismantled, there is no mortar. Discrepancies, errors, and gaps were impossible to resolve because or this zero-tolerance approach. In order to ensure completion on site in difficult locations, a manual band saw handled the work of removing collision material on the backside of the problematic units. This site carving did not affect the front edge of the units, but it did produce a gap where the voussoir surfaces were not coincidental. This happy accident aligns precisely with the Inca wedge [13] process where masons would fill from the backside of a wall with mortar into a voided wedge between stones, while the front and architectural face appeared to be mortar-less. There is room for further exploration to capitalize on the potential of the Inca wedge method.

Conclusion

La Voûte de LeFevre demonstrates the potential of informing contemporary fabrication methodologies with past knowledge surrounding volume. It successfully employs physics simulation to ensure stability through volumetric calculations that serve in reciprocity with volumetric making processes. While aggregate baltic birch plywood serves as an analog, we see potential in other volumetric materials such as autoclave aerated concrete, plaster, or stone.

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